

# Method of moments and time domain analyses of waveguide-based hybrid multiple device oscillators

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**Abstract:** The time-domain analysis and measured performance of a novel multiple oscillator circuit consisting of densely integrated active devices and planar antennas mounted in rectangular waveguide is presented. We detail a prototype seven element circuit using hybrid pHEMT oscillators delivering 220mW at 12.40 GHz.

## 1 Introduction

The analysis and measured performance of a novel multiple-device oscillator with millimetre-wave and MMIC potential is presented. This new circuit, shown in Figure 1, is characterised by a high density of active devices and radiating elements necessitating rigorous electromagnetic and time domain analyses for accurate characterisation. A full-wave method of moments analysis of the waveguide-housed multiple device oscillator is detailed in this paper together with the time domain analysis from the onset of oscillation to steady-state. Our simulation shows whether or not synchronous power combining occurs for a given array design.

The full design procedure is depicted in Figure 2. An electromagnetic analysis is first carried out for a candidate design to determine the antenna self and mutual couplings. This information is then passed to a time-domain simulator using a non-linear oscillator model for the calculation of both the transient and steady-

state behaviour of the array and the possible oscillator modes. The time-domain simulator has been used to determine methods which ensure the survival of only the power-combining mode. To our knowledge this analysis represents the first attempt to investigate rigorously possible limits upon the density of active devices in radiative power combiners and methods for achieving stable power combining in the presence of strong antenna-to-antenna coupling.

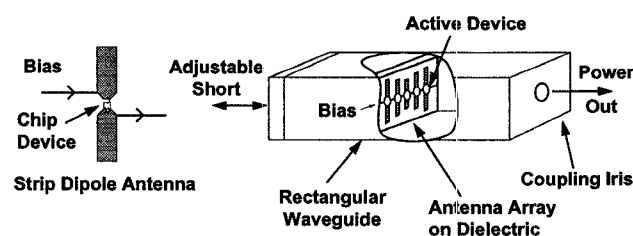


Figure 1: *Rectangular waveguide multiple-device oscillator*

The strong, global coupling between the individual oscillators forces injection locking to a common frequency whilst the correct phase at the output port for power-combining comes about through control of the antenna mutual couplings. The tightly coupled nature of this circuit requires that full account is taken of the antenna interactions. This is in contrast with other studies of power combining of this sort which ignore

the mutual interaction between radiating elements [1].

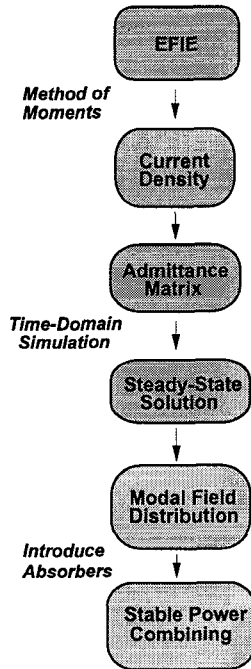


Figure 2: Analysis methodology for stable power combining

## 2 Theoretical Analysis

### 2.1 Electromagnetics

The electric field arising from a delta-function current source is the Green's function  $\bar{G}(r|r')$ ; the solution of the vector Helmholtz equation subject to the waveguide boundary conditions, field matching conditions at the dielectric interfaces and the radiation condition. The total field at any point in the waveguide is the superposition (assuming linear media) of the total elemental current source field and is commonly written as,

$$E(r) = -j\omega\mu_0 \int_V \bar{G}(r|r') J(r') dV' \quad (1)$$

where  $e^{j\omega t}$  time dependence is assumed. The method of moments has been used with piece-wise sinusoidal

and pulse basis functions to solve equation (1) for a strip dipole antenna array located in rectangular waveguide and shown in Figure 1. This solution is then substituted into the original integral equation to determine  $E(r)$  and  $H(r)$  anywhere in the combiner for known complex excitation at each antenna port.

### 2.2 Time-Domain Analysis

The time-domain approach of York [2] has been adopted with modification for the analysis of the coupled oscillator array. This approach assumes near-sinusoidal oscillations of the form  $A(t)e^{j(\omega t + \phi(t))}$  where both  $A(t)$  and  $\phi(t)$  are both "slowly varying" with respect to  $\omega$  so that a frequency-domain description of the coupling network (such as that generated by the method of moments) can be used. A mathematical description of the oscillators is used which approximates the nonlinear variation of device admittance with amplitude as well as predicting the free-running output frequency and output power. The oscillator admittance  $Y_{osc}(\omega, |V|)$  takes the form,

$$Y_{osc} = Y_0 + Y_V |V|^2 + Y_\omega \Delta\omega + Y_C \Delta\omega |V|^2 + Y_{\omega^2} \Delta\omega^2 \quad (2)$$

where  $\Delta\omega = \omega - \omega_0$  and  $\omega_0$  is the free-running frequency of oscillation and  $Y_0 = -G_0 + jB_0$ . The equivalent circuit is determined by fitting, using a least squares technique, a non-linear one-port admittance model to oscillator load pull data generated in MDS with a Hewlett-Packard Root model for the  $0.25 \mu\text{m}$  pHEMT with associated terminations [3]. The analysis leads to a set of rate equations in amplitude and phase at each port of the array which are integrated numerically using a modified Runge-Kutta algorithm [4]. This approach has been successful in describing some of the more complex behaviour observed in these coupled oscillators, notably the spectrum when the oscillators are close to locking, the behaviour of the array under the influence of an external locking signal and a tolerable spread in free-running frequencies of the oscillator array for purely synchronous behaviour.

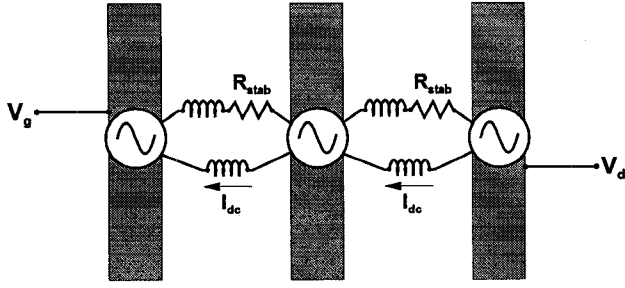


Figure 3: *Lumped element coupling for stable power combining*

The time domain simulation also indicates whether a particular design results in stable power combining. A multiple device oscillator of this type with purely electromagnetic coupling between the antennas can evolve to the locked state. However, some means to ensure survival of the power combining (in-phase) mode must be undertaken. We have adopted an approach which mixes both electromagnetic and lumped element coupling with a stabilising resistor [5] to force the circuit into the power combining mode. Figure 3 shows the approach.

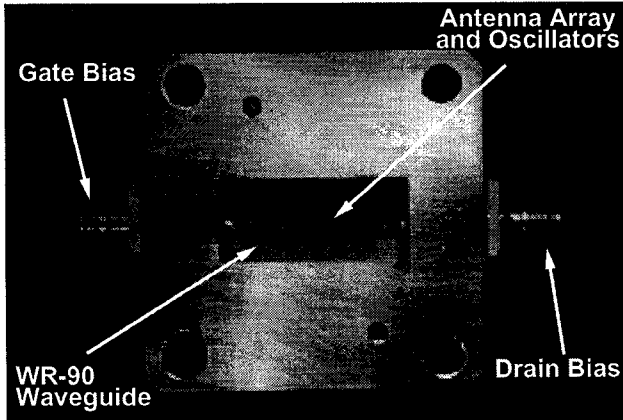


Figure 4: *Waveguide test fixture with seven device oscillator*

### 3 Results

To illustrate the method we consider the seven device oscillator, fabricated on 0.010in  $\epsilon_r=10.5$ , and shown in Figure 4. The oscillators themselves are described mathematically by Equation (2) and physically realised through 0.25  $\mu\text{m}$  gate length pHEMTs (total gate width 2.4 mm) with appropriate terminations to produce oscillation at 12 GHz. Figure 5 shows the implemented circuit; the inductors are realised as bond wires and the capacitor is an ATC millimetre-wave part.

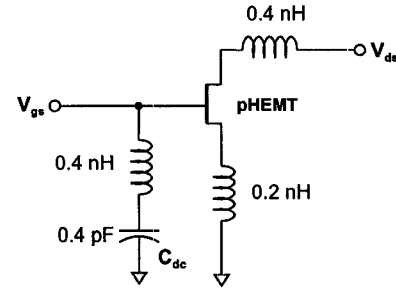


Figure 5: *HEMT oscillator circuit*

The time domain analysis is used to predict the oscillation frequency, the locked amplitudes and the output power from the circuit. Figure 6 shows the amplitudes and frequencies of four of the seven oscillators (the other three can be determined from symmetry) as the oscillations build from noise. A steady amplitude ( $\frac{dA_i}{dt} = 0$ ) is indicative of sinusoidal oscillation at each port. The oscillation frequency is normalised to 12 GHz and the amplitude to 1 V. Clearly evident is the evolution to the locked state and the associated locked frequency and amplitudes. The simulation predicts oscillation at 10.85 GHz and 340 mW into a matched waveguide load. Our measurements show stable, single-frequency oscillation at 12.40 GHz with 220 mW of output power which should be considered good agreement. The close-in spectrum is shown in Figure 7. It is in the region of that expected from a HEMT oscillator at this frequency but can be improved through locking to an external, low-noise reference.

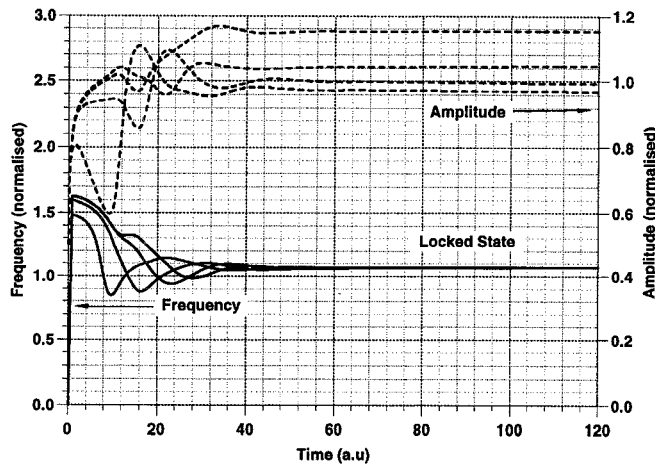


Figure 6: Time-domain simulation showing start-up transient and evolution to the locked state for four oscillators in a seven device oscillator

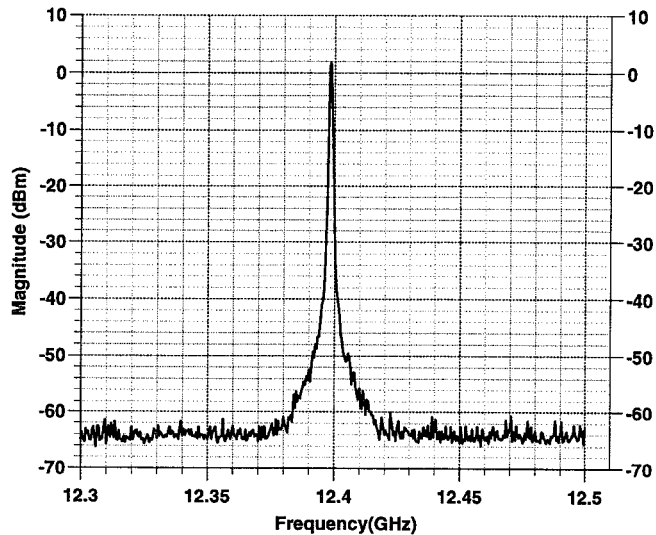


Figure 7: Measured close-in spectrum of the seven pHEMT multiple device oscillator

## 4 Conclusion

The design methodology and successful implementation of a novel class of multiple devices oscillators has been presented. A full wave method of moments analysis together with time domain simulation describes well the observed behaviour of the circuit. We have achieved 220mW from a seven device oscillator at 12.40. The high packing-density of devices in these circuits suggests that a MMIC implementation will be commercially viable.

## 5 Acknowledgements

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## References

- [1] Zoya Popović et al. "A 100-MESFET grid oscillator". *IEEE Trans. Microwave Theory Tech.*, 39(2):193–199, 1991.
- [2] Robert A. York et al. "Oscillator array dynamics with broadband N-port coupling networks". *IEEE Trans. Microwave Theory Tech.*, 42(11):2040–2045, 1994.
- [3] Katsumi Fukumoto et al. "Mathematical representation of microwave oscillator characteristics by use of the Rieke diagram". *IEEE Trans. Microwave Theory Tech.*, 31(11):954–959, 1983.
- [4] William Press et al. *Numerical Recipes in C*. Cambridge University Press, 2nd edition, 1992.
- [5] Shigeji Nogi et al. "Mode analysis and stabilization of a spatial power combining array with strongly coupled oscillators". *IEEE Trans. Microwave Theory Tech.*, 41(10):1827–1837, 1993.